

# Proposal of demonstration of the Goldbach's conjecture by Rémy Aumeunier

The Goldbach's conjecture is the unproven mathematic assertion which states : All whole even number greater than 2 can be written as the sum of two prime numbers. It was formulated in 1742 by Christian Goldbach, it is one of the oldest unsolved problem of the theorie of number and mathematics.

## 1 Preamble

We will agree to say that at the time of I writing proposal of demonstration, Goldbach's conjecture is not proven, that it is verified for all even integers less than  $4.10^{18}$ . And that we don't know why some primes, decompose the even integers in sums of two primes.

## 2 Installing the tools

In this article to prove the conjecture, I propose to extend the decomposition of integers into prime, by introducing all eligible primes to the decomposition of the even integer considered, with  $p_n < n$ .

$$n \rightarrow \begin{pmatrix} (n) \bmod(2) = \dots \\ (n) \bmod(3) = \dots \\ (n) \bmod(5) = \dots \\ (n) \bmod(7) = \dots \\ \dots \\ (n) \bmod(p_n) = \dots \end{pmatrix} = \text{Signature}_n \square$$

From now on, and in the rest of the proposal, I will only take into account the result vector.

$$\text{Sgn}_n = \square$$

### 2.1 Minimalist analysis

I can say that the signature is unique, owing to the signature extends or encompasses the prime factor decomposition of the integer.

I can say that if there is no zero in the signature, then  $n$  is a prime, owing the signature has all the primes eligible to the decomposition of  $n$ , furthermore we know that a composite number always has a factor smaller than its square root.

### 3 Proposed proof of the Goldbach conjecture

In this first part I present the underlying notion at the origin of the demonstration. To break down an even integer in sums of two prime numbers, it is enough to decompose in sums the values of the signature of  $2n$  without introducing zero, then to calculate the integer associated has each signature thus created. As all the integers are decomposable in sums of 2 elements different zero. There is always a solution for  $2n = p_a + p_b$ .

#### 3.1 Exemple

$$Sgn_{2n} = \begin{pmatrix} (2n) \bmod(2) = 0 = 1 + 1 \\ (2n) \bmod(3) = \dots = a_3 + b_3 \\ (2n) \bmod(5) = \dots = a_5 + b_5 \\ (2n) \bmod(7) = \dots = a_7 + b_7 \\ \dots \end{pmatrix}$$

So to this decomposition or to its two signatures, I could associate two integers.

$$Sgn_a[] = p_a \quad , \quad Sgn_b[] = p_b$$

If there is no zero present in  $Sgn_a[]$   $Sgn_b[]$ , then these integers will be prime, this decomposition method is always possible regardless of the value of  $2n$ , and this method allows to justify the all solutions which decompose  $2n$  in sums. It is just necessary not to introduce of zero in  $Sgn_a[]$   $Sgn_b[]$  at the levels the factors less than the square root when one decomposes  $Sgn_{2n}[]$ .

#### 3.2 Proposed proof

So if I consider the signature of a non-decomposable even integer as a sums of two prime numbers. This implies that all primes less than  $2n$  modify the signature of  $2n$  by imposing a zero at the lower factor levels of the square root of  $2n$ ,  $(2n - p_{eligible}) \bmod(p_x) = 0$   $p_x < \sqrt{2n}$ , which means that all eligible primes have a common element in their signature with  $2n$ , and it is trivially impossible owing all prime numbers have a different signature, cf. Dirichlet's theorem on arithmetic progressions and  $\pi(2n) \gg \pi(\sqrt{2n})$

### 3.3 Decomposition method

To calculate the value of the integer from its signature one can use the arithmetic associated with the representation of integers in primorial base.

$$\begin{aligned}
 n = & 1 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 2 \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + 2 \cdot 3 \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} + 2 \cdot 3 \cdot 5 \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} + 2 \cdot 3 \cdot 5 \cdot 7 \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{bmatrix} + \dots \\
 & \underbrace{\hspace{1.5cm}}_{(n) \bmod(2)} \quad \underbrace{\hspace{2.5cm}}_{(n) \bmod(3)} \quad \underbrace{\hspace{3.5cm}}_{(n) \bmod(5)} \quad \underbrace{\hspace{4.5cm}}_{(n) \bmod(7)} \quad \dots \\
 & \underbrace{\hspace{10cm}}_{\dots(11)}
 \end{aligned}$$

To decompose  $2n$ , I can also look for a solution by construction so for  $2n = 98$   $\sqrt{98} = 9.89\dots$ . I must therefore construct 2 integers without zero at the levels of the prime factors 2, 3, 5, 7 of the signature of  $Sgn_{98}$ . Because we know that a compound number always has a factor smaller than its square root.

$$\begin{aligned}
 p_a &= 1 \cdot [1] & p_a &= 1 \cdot [1] + 2 \cdot [0] = 1 \\
 (98) \bmod(2) &= \mathbf{0} = \mathbf{1+1} & (98) \bmod(2) &= \mathbf{0} = \mathbf{1+1} \\
 (98) \bmod(3) &= \mathbf{2} = \mathbf{1+1} & (98) \bmod(3) &= \mathbf{2} = \mathbf{1+1} \\
 (98) \bmod(5) &= \mathbf{3} = \mathbf{1+2} & (98) \bmod(5) &= \mathbf{3} = \mathbf{1+2} \\
 (98) \bmod(7) &= \mathbf{0} = \mathbf{1+6} & (98) \bmod(7) &= \mathbf{0} = \mathbf{1+6} \\
 (98) \bmod(11) &= \mathbf{10} = \mathbf{1+9} & (98) \bmod(11) &= \mathbf{10} = \mathbf{1+9} \\
 \dots & & \dots &
 \end{aligned}$$

$$\begin{aligned}
 p_a &= 1 \cdot [1] + 2 \cdot [0] + 2 \cdot 3 \cdot [1] = 7 & p_a &= 1 \cdot [1] + 2 \cdot [0] + 2 \cdot 3 \cdot [1] + 2 \cdot 3 \cdot 5 \cdot [1] = 37 \\
 (98) \bmod(2) &= \mathbf{0} = \mathbf{1+1} & (98) \bmod(2) &= \mathbf{0} = \mathbf{1+1} \\
 (98) \bmod(3) &= \mathbf{2} = \mathbf{1+1} & (98) \bmod(3) &= \mathbf{2} = \mathbf{1+1} \\
 (98) \bmod(5) &= \mathbf{3} = \mathbf{2+1} & (98) \bmod(5) &= \mathbf{3} = \mathbf{2+1} \\
 (98) \bmod(7) &= \mathbf{0} = \mathbf{0+0} & (98) \bmod(7) &= \mathbf{0} = \mathbf{2+5} \\
 (98) \bmod(11) &= \mathbf{10} = \mathbf{7+3} & (98) \bmod(11) &= \mathbf{10} = \mathbf{4+6} \\
 \dots & & \dots &
 \end{aligned}$$

The addition of Primorial preserves the previous values and allows to build step by step the searched values. It is useless to go beyond the square root, here :  $98 = 37 + 61$

### 3.4 Arithmetic remark

I only found 83 integers, so the largest has a for value of 63274 such that  $2n = p_a + p_b$  with  $p_b > \sqrt{2n}$ , so a prime number less than  $\sqrt{2n}$  is sufficient to

decompose  $2n$ , which corroborates Chen Jingrun's theorem.

$$63274 = 293 + 62981 \quad \sqrt{63274} = 251.54\dots$$

## 4 Corollary

During the international congress of mathematicians of 1912 in Cambridge, Edmund Landau presented 4 problems which are linked, in this party I propose to apprehend the conjecture of the twin prime numbers and the legend conjecture.

### 4.1 There is an infinite number of twin primes.

The quantity of elements present in the signature is decorrelated, of the difference between 2 twin primes, triplets, ... owing to. I can add the elements that show that there are an infinite number of twin primes.

$$(p_a) \bmod (2 \cdot 3 \cdot 5 \cdot 7 \cdots) = p_b \quad , \quad (p_a) \bmod (2 \cdot 3 \cdot 5 \cdot 7 \cdots) = p_b \cdot n$$

with the Primorial  $\prod p_n \ll p_a$ .

### 4.2 Legendre's conjecture

There exists a prime between  $n^2$  and  $(n+1)^2$  for all integers  $n \geq 1$ , since  $2n = p_a + p_b$  with  $p_b < \approx \sqrt{2n}$  then :

$$(n+1)^2 \pm \{0, 1\} - p_b = p_a \quad p_a < (n+1)^2, p_a > n^2$$

## 5 $\pi(n)$

From the signature of  $n$ , I propose to calculate the quantity of combination without zero, to know the value of  $\pi(n)$ , all the difficulty lies in the upper bound of the product, which remains to be established.

$$Sgn_n = \left( \begin{array}{l} (n) \bmod (2) \in \{0, 1\} \\ (n) \bmod (3) \in \{0, 1, 2\} \\ (n) \bmod (5) \in \{0, 1, 2, 3, 4\} \\ (n) \bmod (7) \in \{0, 1, 2, 3, 4, 5, 6\} \\ \dots \\ (n) \bmod (p_n) \in \{0, 1, 2, \dots, p_{(n-1)}\} \\ \dots \end{array} \right) \pi(n) = \prod_{p=2}^{p=\dots} (p-1)$$

## 6 Conclusion

If you do not admit that the conjecture is proven, you can already say that you know how to justify why some primes do or do not decompose the even integers.

Thanks for your attention.

## Références

- [1] Conjecture de Goldbach.
- [2] Conjecture de Legendre.
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